Unit 8, Part 1

Linked Lists

Computer Science S-111
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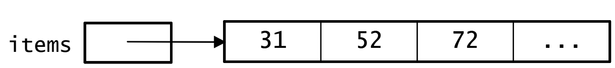
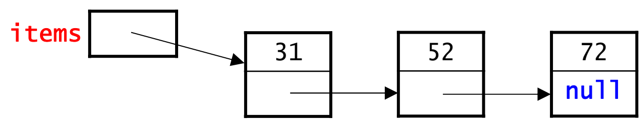
Representing a Sequence of Data

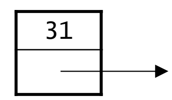
- Sequence – an ordered collection of items (position matters)
  - we will look at several types: lists, stacks, and queues
- Most common representation = an array
- Advantages of using an array:
  - easy and efficient access to *any* item in the sequence
    - `items[i]` gives you the item at position i in O(1) time
    - known as *random access*
  - very compact (but can waste space if positions are empty)
- Disadvantages of using an array:
  - have to specify an initial array size and resize it as needed
  - inserting/deleting items can require shifting other items
    - ex: insert 63 between 52 and 72



Alternative Representation: A Linked List



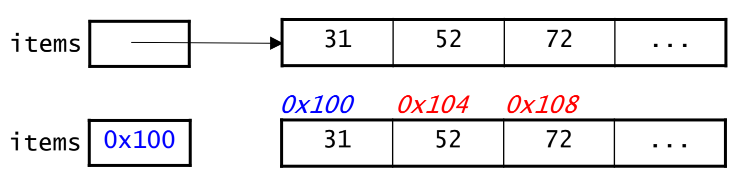
- A linked list stores a sequence of items in separate *nodes*.
- Each node is an object that contains:
  - a single item
  - a "link" (i.e., a reference) to the node containing the next item



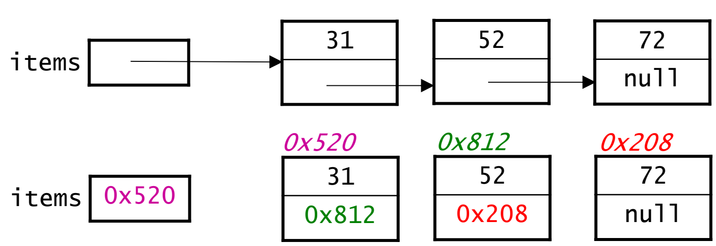
- The last node in the linked list has a link value of `null`.
- The linked list as a whole is represented by a variable that holds a reference to the first node.
  - e.g., `items` in the example above

Arrays vs. Linked Lists in Memory

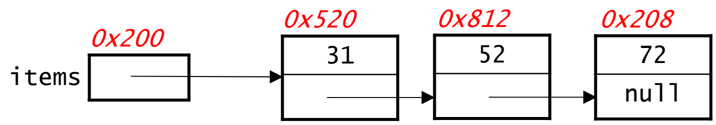
- In an array, the elements occupy consecutive memory locations:



- In a linked list, the nodes are distinct objects.
  - do *not* have to be next to each other in memory
  - that's why we need the links to get from one node to the next!



Linked Lists in Memory

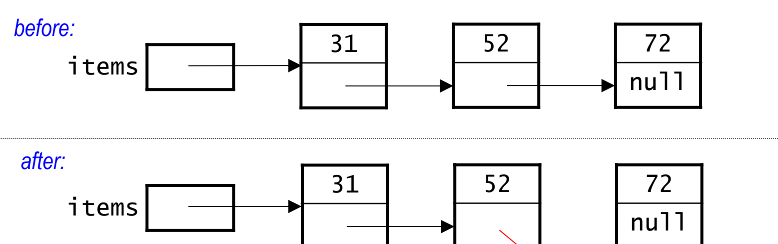


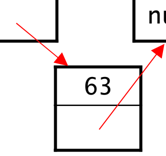
- Here's how the above linked list might actually look in memory:

0x200	0x520	← *the variable* `items`
0x204		
0x208	72	*the last node*
0x212	null	
0x216		
…	…	
0x520	31	*the first node*
0x524	0x812	
0x528		
…	…	
0x812	52	*the second node*
0x816	0x208	

Features of Linked Lists

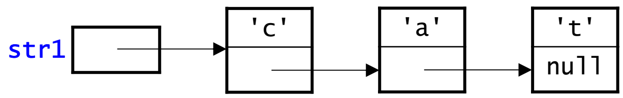
- They can grow without limit (provided there is enough memory).
- Easy to insert/delete an item – no need to "shift over" other items.
  - for example, to insert 63 between 52 and 72:





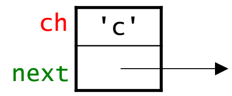
- Disadvantages:
  - they don't provide random access
    - need to "walk down" the list to access an item
  - the links take up additional memory

A String as a Linked List of Characters



- Each node represents one character.

- Java class for this type of node:




```
public class StringNode {
    private char ch;
    private StringNode next;

    public StringNode(char c, StringNode n) {
        this.ch = c;
        this.next = n;
    }
    ...
}
```

same type as the node itself!

- The string as a whole is represented by a variable that holds a reference to the node for the first character (e.g., `str1` above).

A String as a Linked List (cont.)

- An empty string will be represented by a null value.

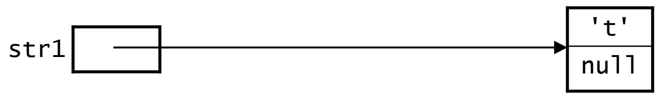
  example:
  `StringNode str2 = null;`

- We will use *static* methods that take the string as a parameter.
  - e.g., we'll write `length(str1)` instead of `str1.length()`
  - outside the class, call the methods using the class name:
    `StringNode.length(str1)`

- This approach allows the methods to handle empty strings.
  - if `str1 == null`:
    - `length(str1)` will work
    - `str1.length()` will throw a `NullPointerException`

Building a Linked List of Characters I

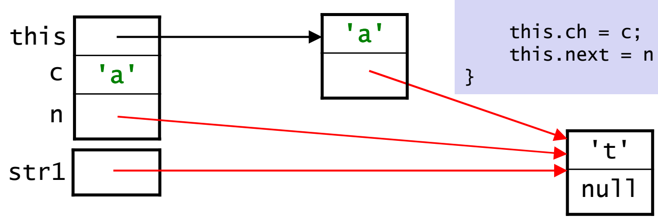
```
public StringNode(char c,
                  StringNode n) {
    this.ch = c;
    this.next = n;
}
```



- We can use the `StringNode` constructor to build the linked list from the previous slide.
- One way is to start with the last node and work towards the front:

```
StringNode str1 = new StringNode('t', null);
```

Building a Linked List of Characters II



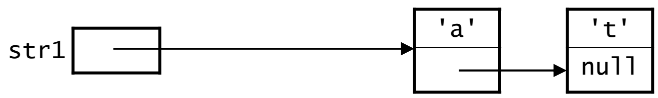

```
public StringNode(char c,
                  StringNode n) {
    this.ch = c;
    this.next = n;
}
```

- We can use the `StringNode` constructor to build the linked list from the previous slide.
- One way is to start with the last node and work towards the front:

```
StringNode str1 = new StringNode('t', null);
str1 = new StringNode('a', str1);
```

Building a Linked List of Characters III

```
public StringNode(char c,
                  StringNode n) {
    this.ch = c;
    this.next = n;
}
```

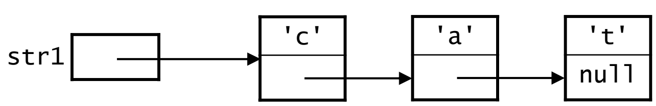


- We can use the `StringNode` constructor to build the linked list from the previous slide.
- One way is to start with the last node and work towards the front:

```
StringNode str1 = new StringNode('t', null);
str1 = new StringNode('a', str1);
```

Building a Linked List of Characters IV

```
public StringNode(char c,
                  StringNode n) {
    this.ch = c;
    this.next = n;
}
```



- We can use the `StringNode` constructor to build the linked list from the previous slide.
- One way is to start with the last node and work towards the front:

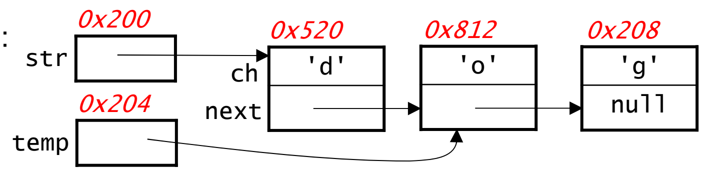
```
StringNode str1 = new StringNode('t', null);
str1 = new StringNode('a', str1);
str1 = new StringNode('c', str1);
```

- Later, we'll see methods that can be used to build a linked list and add nodes to it.

Review of Variables

- A variable or variable expression represents both:
  - a "box" or location in memory (the *address* of the variable)
  - the contents of that "box" (the *value* of the variable)

- Practice:

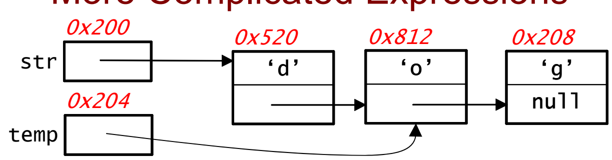



```
StringNode str;   // points to the first node
StringNode temp;  // points to the second node
```

expression	*address*	*value*
`str`	0x200	0x520 (ref to the 'd' node)
`str.ch`		
`str.next`		

Assumptions:
- `ch` *field has the same memory address as the node itself.*
- `next` *field comes 2 bytes after the start of the node.*

More Complicated Expressions

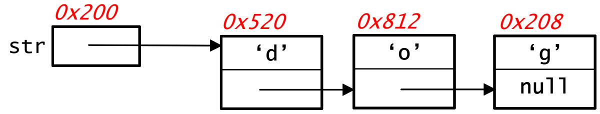


- Example: `temp.next.ch`

- Start with the beginning of the expression: `temp.next`
  It represents the `next` field of the node to which `temp` refers.
  - address =
  - value =

- Next, consider `temp.next.ch`
  It represents the `ch` field of the node to which `temp.next` refers.
  - address =
  - value =

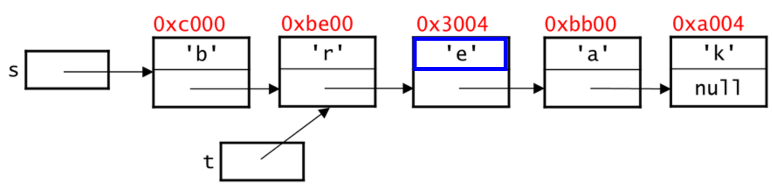
What are the address and value of `str.next.next`?



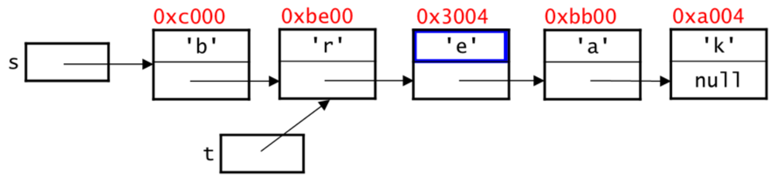
- `str.next` is…

- thus, `str.next.next` is…

What expression using `t` would give us `'e'`?



What expression using `t` would give us `'e'`?



Working backwards…

- I know that I need the `ch` field in the `'e'` node
- Where do I have a reference to the 'e' node?
- What expression can I use for the box containing that reference?

Review of Assignment Statements

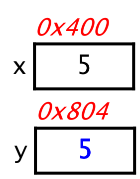
- An assignment of the form

  `var1 = var2;`

  - takes the value inside `var2`
  - copies it into `var1`

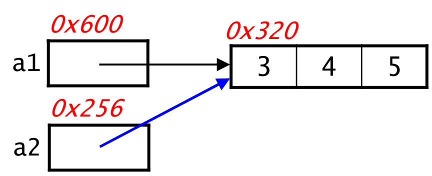
- Example involving integers:

```
int x = 5;
int y = x;
        5
```

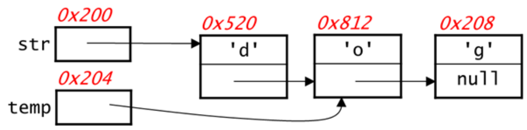


- Example involving references:

```
int[] a1 = {3, 4, 5};
int[] a2 = a1;
           0x320
```



What About These Assignments?



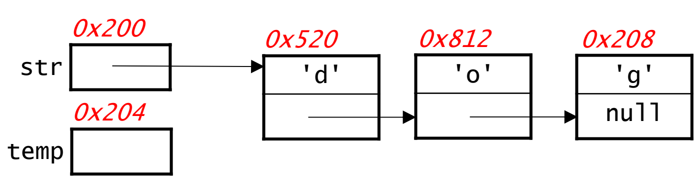
- *Identify the two boxes.*
- *Determine the value in the box specified by the right-hand side.*
- *Copy that value into the box specified by the left-hand side.*

1) `str.next = temp.next;`

2) `temp.next = temp.next.next;`

Writing an Appropriate Assignment

- If `temp` didn't already refer to the `'o'` node, what assignment would be needed to make it refer to that node?



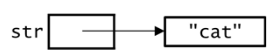
  - start by asking: where do I currently have a reference to the `'o'` node?
  - then ask: what expression can I use for that box?
  - then write the assignment:

A Linked List Is a Recursive Data Structure!

- Recursive definition: a linked list is either
  a) empty or
  b) a single node, followed by a linked list
- Viewing linked lists in this way allows us to write recursive methods that operate on linked lists.

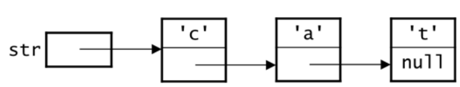
Recursively Finding the Length of a String

- For a Java `String` object:




```
public static int length(String str) {
    if (str.equals("")) {
        return 0;
    } else {
        int lenRest = length(str.substring(1));
        return 1 + lenRest;
    }
}
```

- For a linked-list string:




```
public static int length(StringNode str) {
    if (str == null) {
        return 0;
    } else {
        int lenRest = length(str.next);
        return 1 + lenRest;
    }
}
```

An Alternative Version of the Method

- Original version:

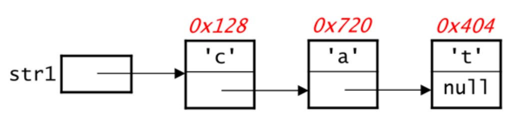
```
public static int length(StringNode str) {
    if (str == null) {
        return 0;
    } else {
        int lenRest = length(str.next);
        return 1 + lenRest;
    }
}
```

- Version without a variable for the result of the recursive call:

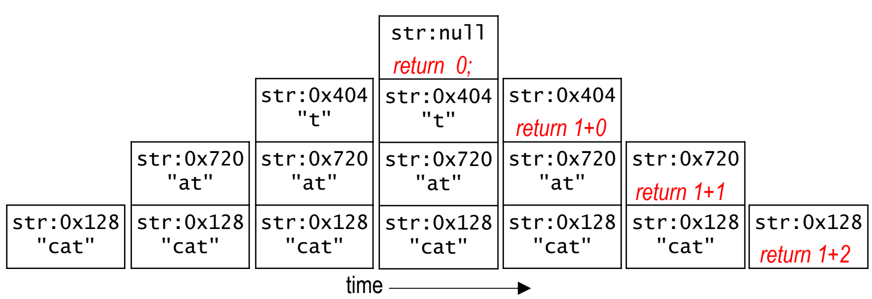
```
public static int length(StringNode str) {
    if (str == null) {
        return 0;
    } else {
        return 1 + length(str.next);
    }
}
```

Tracing `length()`

```
public static int length(StringNode str) {
    if (str == null) {
        return 0;
    } else {
        return 1 + length(str.next);
    }
}
```

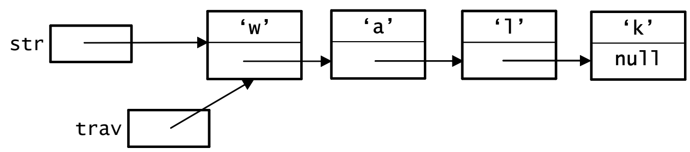


- Example: `StringNode.length(str1)`



Using Iteration to Traverse a Linked List

- Many tasks require us to traverse or "walk down" a linked list.
- We just saw a method that used recursion to do this.
- It can also be done using iteration (for loops, while loops, etc.).
- We make use of a variable (call it `trav`) that keeps track of where we are in the linked list.

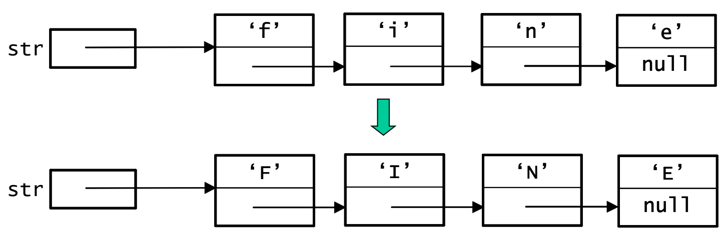


- Template for traversing an entire linked list:

```
StringNode trav = str;     // start with first node
while (trav != null) {
    // process the current node here
    trav = trav.next;      // move trav to next node
}
```

Example of Iterative Traversal

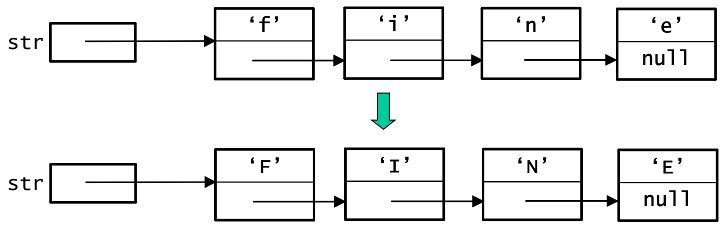
- `toUpperCase(str)`: converting `str` to all upper-case letters



- Similar to the built-in method for Java `String` objects.
- This method processes linked-list strings:
  - uses a loop to process one `StringNode` at a time
  - modifies the internals of the string (unlike the built-in version)
  - thus, it doesn't need to return anything

Example of Iterative Traversal (cont.)

- `toUpperCase(str)`: converting `str` to all upper-case letters

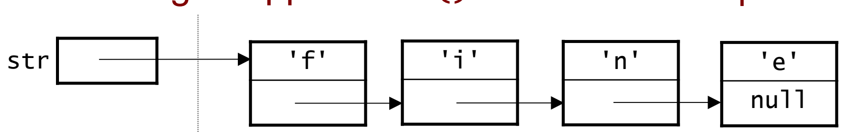


- Here's the method:

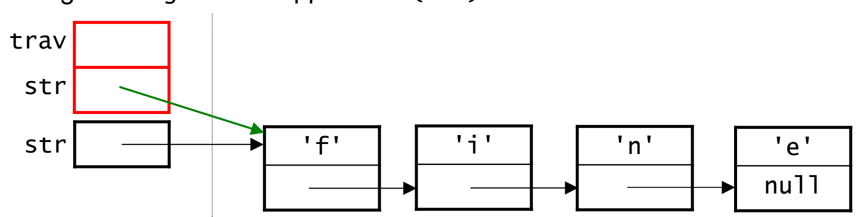
```
public static void toUpperCase(StringNode str) {
     StringNode trav = str;
     while (trav != null) {
          trav.ch = Character.toUpperCase(trav.ch);
          trav = trav.next;
     }
}
```

  - uses a built-in static method from the `Character` class to convert a single `char` to upper case

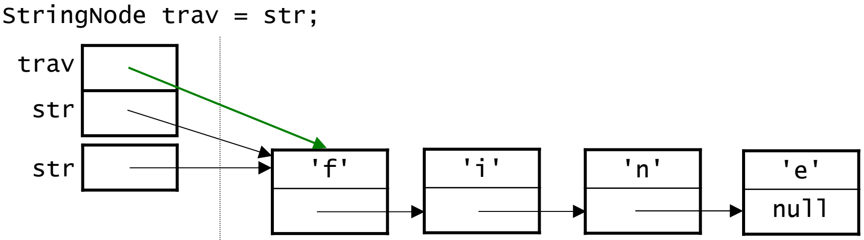
Tracing `toUpperCase()`: Before the Loop



Calling `StringNode.toUpperCase(str)` adds a stack frame to the stack:



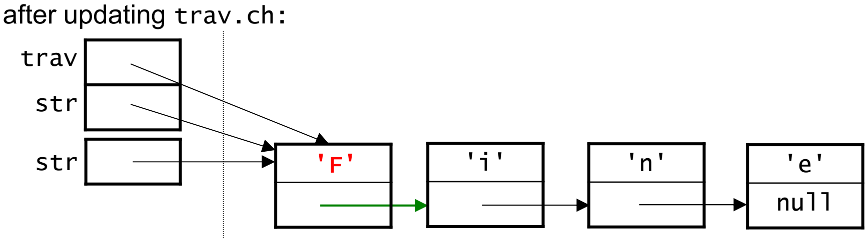
`StringNode trav = str;`



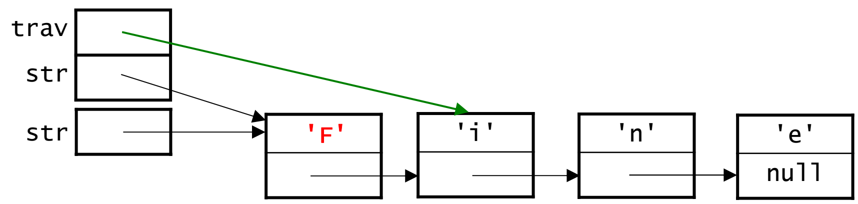
Tracing `toUpperCase()`: First Iteration of Loop

```
while (trav != null) {
    trav.ch = Character.toUpperCase(trav.ch);
    trav = trav.next;
}
```

after updating `trav.ch`:



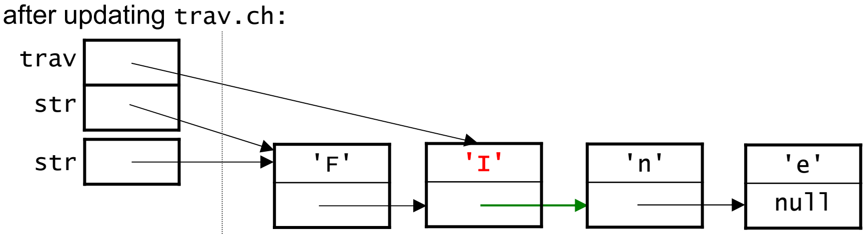
after updating `trav`:



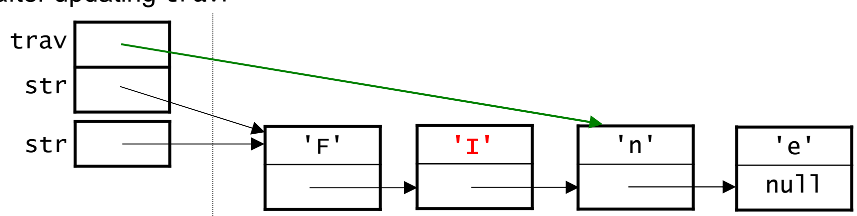
Tracing `toUpperCase()`: Second Iteration

```
while (trav != null) {
    trav.ch = Character.toUpperCase(trav.ch);
    trav = trav.next;
}
```

after updating `trav.ch`:



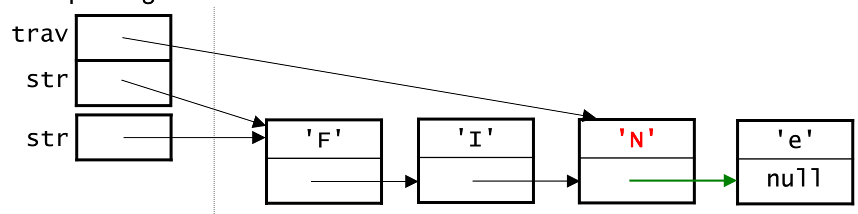
after updating `trav`:



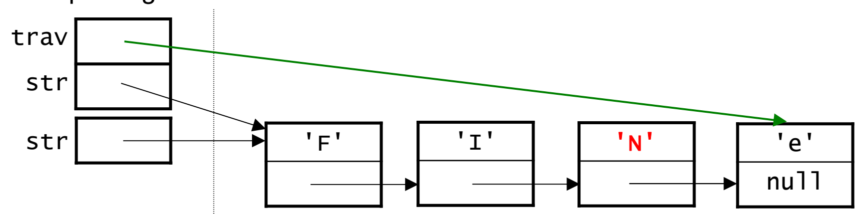
Tracing `toUpperCase()`: Third Iteration

```
while (trav != null) {
    trav.ch = Character.toUpperCase(trav.ch);
    trav = trav.next;
}
```

after updating `trav.ch`:



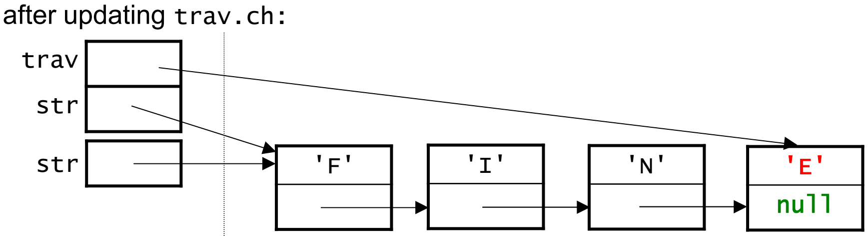
after updating `trav`:



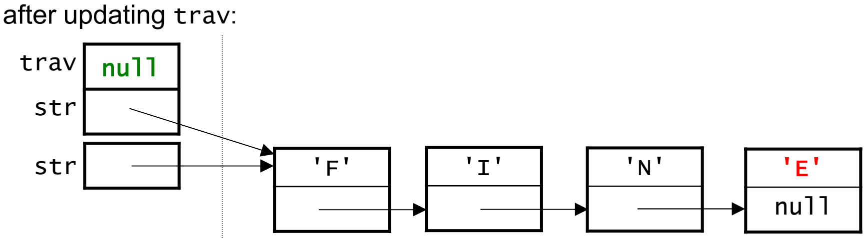
Tracing `toUpperCase()`: Fourth Iteration

```
while (trav != null) {
    trav.ch = Character.toUpperCase(trav.ch);
    trav = trav.next;
}
```

after updating `trav.ch`:



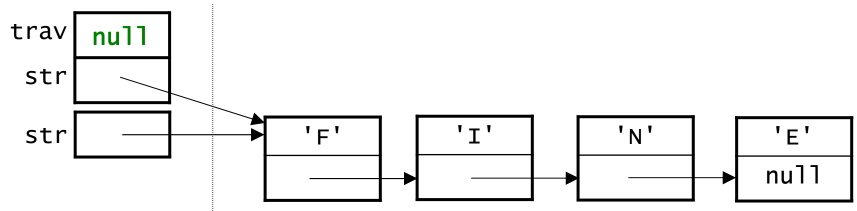
after updating `trav`:



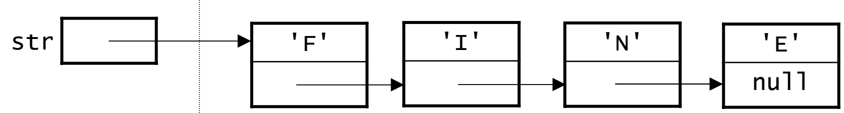
Tracing `toUpperCase()`: Finishing Up

```
while (trav != null) {
    trav.ch = Character.toUpperCase(trav.ch);
    trav = trav.next;
}
```

results of the final iteration:

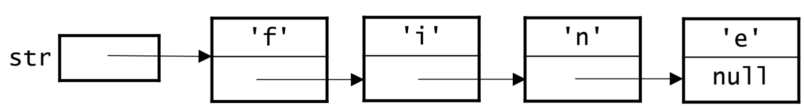


and now `trav == null`, so we end the loop and return:



Getting the Node at Position i in a Linked List

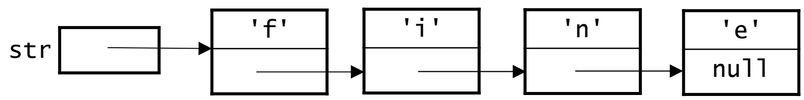
- `getNode(str, i)` – should return a reference to the `i`th node in the linked list to which `str` refers



- Examples:
  - `getNode(str, 0)` should return a ref. to the `'f'` node
  - `getNode(str, 3)` should return a ref. to the `'e'` node
  - `getNode(str.next, 2)` should return a ref. to…?

- More generally, when `0 < i <` *length of list*, `getNode(str, i)` is equivalent to `getNode(str.next, i-1)`

Getting the Node at Position i in a Linked List



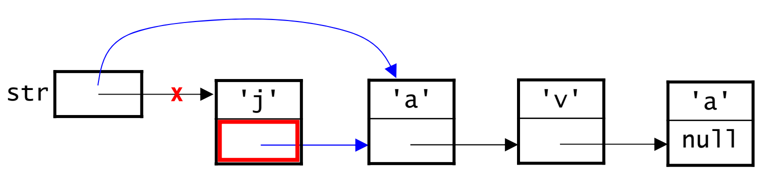
- Recursive approach to `getNode(str, i)`:
  - if `i == 0`, return `str` (base case)
  - else call `getNode(str.next, i-1)` and return what it returns!
  - other base case?
- Here's the method:

```
private static StringNode getNode(StringNode str, int i) {
    if (i < 0 || str == null) { // base case 1: no node i
        return null;
    } else if (i == 0) {        // base case 2: just found
        return str;
    } else {
        return getNode(str.next, i-1);
    }
}
```

Deleting the Item at Position i

- Special case: `i == 0` (deleting the first item)
- Update our reference to the first node by doing:

```
str = str.next;
```



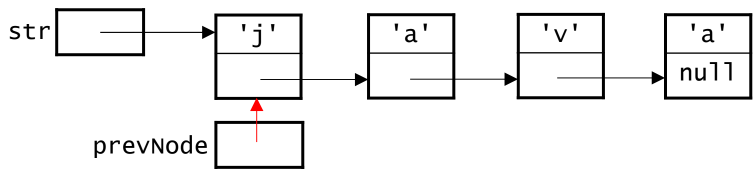
Deleting the Item at Position i (cont.)

- General case: `i > 0`

  1. Obtain a reference to the *previous* node:

```
StringNode prevNode = getNode(i - 1);
```

(example for i == 1)



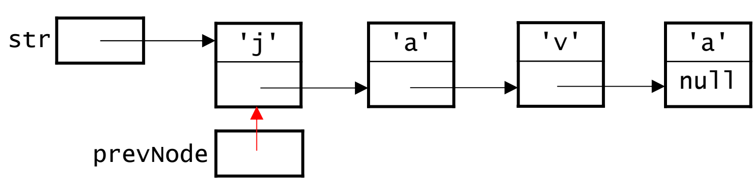
Deleting the Item at Position i (cont.)

- General case: `i > 0`

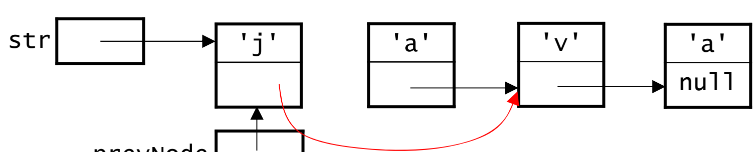
  2. Update the references to remove the node

(example for i == 1)

before:



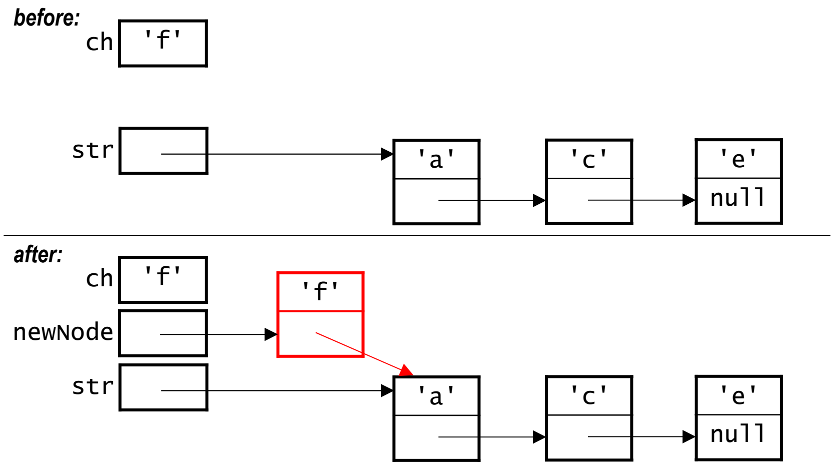
after:



________________ = ______________________;

Inserting an Item at Position i

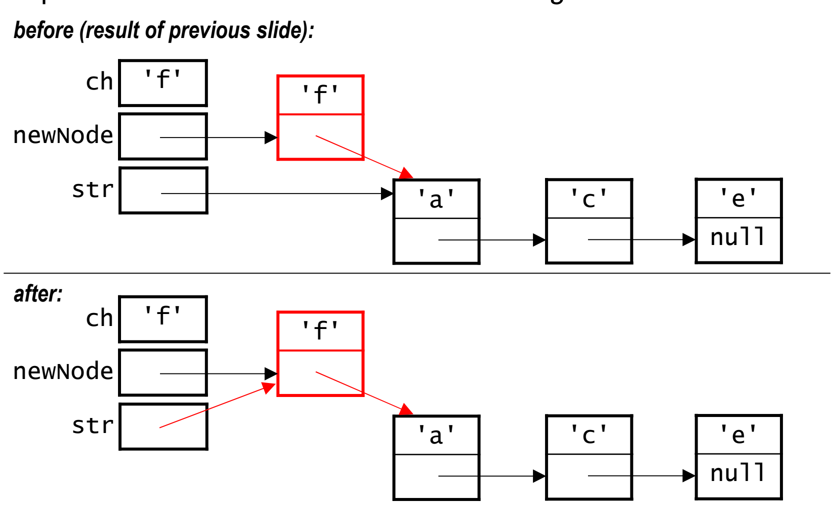
- <u>Special case:</u> `i == 0` (insertion at the front of the list)
- Step 1: *Create* the new node. *Fill in the blanks!*




```
StringNode newNode = new StringNode(_______, _______);
```

Inserting an Item at Position i (cont.)

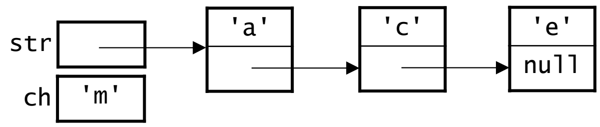
- <u>Special case:</u> `i == 0` (continued)
- Step 2: *Insert* the new node. *Write the assignment!*



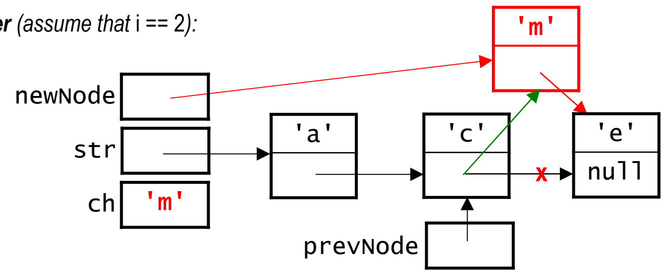
Inserting an Item at Position i (cont.)

- <u>General case:</u> `i > 0` (insert *before* the item currently in posn `i`)

before:



after *(assume that* i == 2*):*




```
StringNode prevNode = getNode(i - 1);
StringNode newNode = new StringNode(ch, ________________);
__________________________________ // one more line
```

Returning a Reference to the First Node

- Both `deleteChar()` and `insertChar()` return a reference to the first node in the linked list. For example:

```
public static StringNode deleteChar(StringNode str, int i) {
    …
    if (i == 0) {               // special case
        str = str.next;
    } else {                    // general case
        StringNode prevNode = getNode(str, i-1);
        if (prevNode != null && prevNode.next != null) {
            prevNode.next = prevNode.next.next;
            …
    }

    return str;
}
```

- Clients should call them as part of an assignment:

```
s1 = StringNode.deleteChar(s1, 0);
s2 = StringNode.insertChar(s2, 0, 'h');
```

- If the first node changes, the client's variable will be updated to point to the new first node.

Creating a Copy of a Linked List

- `copy(str)` – create a copy of *the entire list* to which `str` refers
- Recursive approach:
  - base case: if `str` is empty, return `null`
  - else: – make a recursive call to copy the rest of the linked list
    – create and return a copy of the first node,
    with its next field pointing to the copy of the rest

```
public static StringNode copy(StringNode str) {
    if (str == null) {        // base case
        return null;
    }

    // make a recursive call to copy the rest of the list
    StringNode copyRest = copy(str.next);

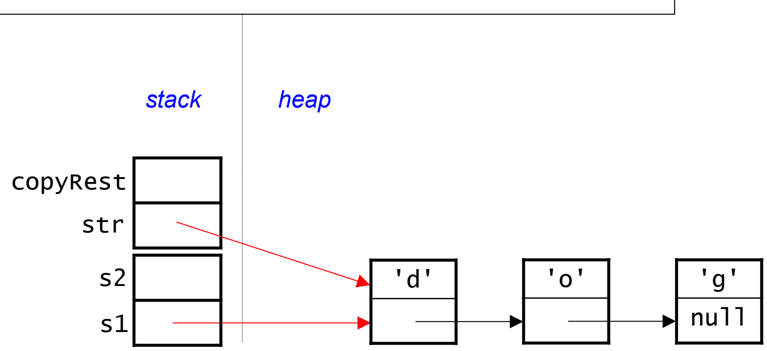
    // create and return a copy of the first node,
    // with its next field pointing to the copy of the rest
    return new StringNode(str.ch, copyRest);
}
```

Tracing copy(): the initial call

- From a client: `StringNode s2 = StringNode.copy(s1);`

```
public static StringNode copy(StringNode str) {
    if (str == null) {
        return null;
    }

    StringNode copyRest = copy(str.next);
    return new StringNode(str.ch, copyRest);
}
```



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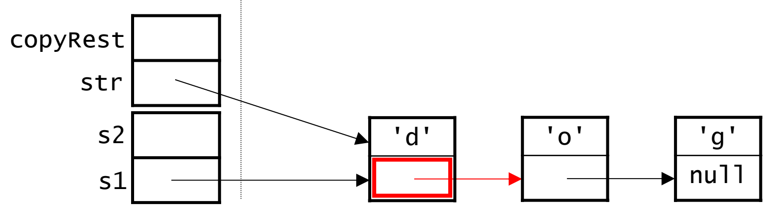
    StringNode copyRest = copy(str.next);
    return new StringNode(str.ch, copyRest);
}
```



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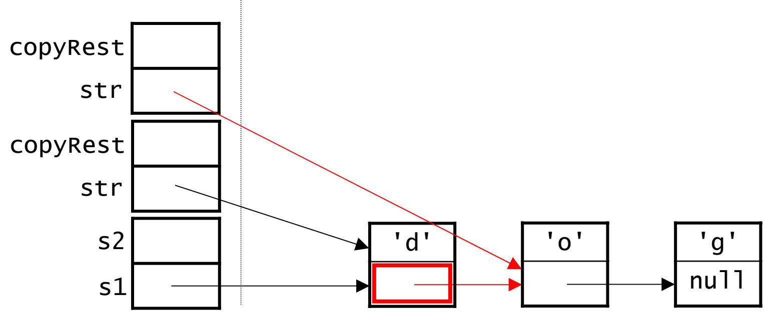

Tracing copy(): the recursive calls

```
public static StringNode copy(...) {
  if (str == null) {
      return null;
  }

  StringNode copyRest = copy(str.next);
  return new StringNode(str.ch,
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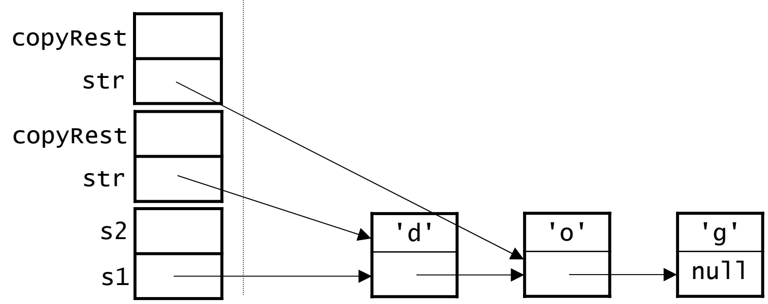
}
```



Tracing copy(): the recursive calls

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  if (str == null) {
      return null;
  }

  StringNode copyRest = copy(str.next);
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}
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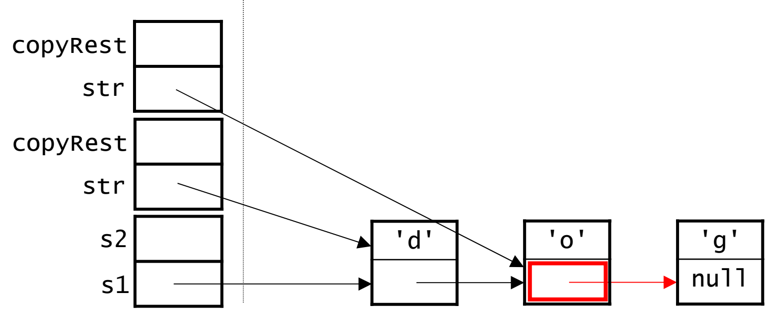

Tracing copy(): the recursive calls

```
public static StringNode copy(...) {
   if (str == null) {
       return null;
   }
   StringNode copyRest = copy(str.next);
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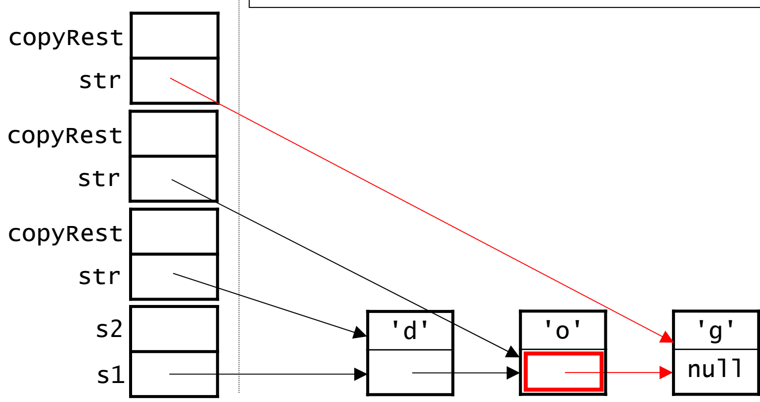
}
```



Tracing copy(): the recursive calls

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       return null;
   }
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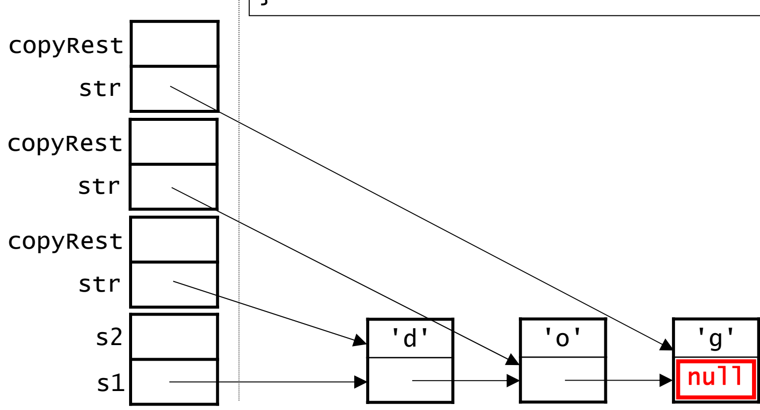

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      return null;
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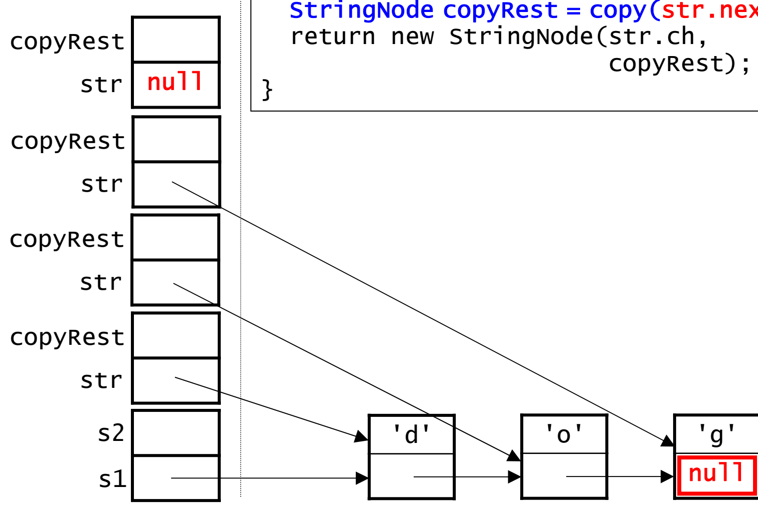
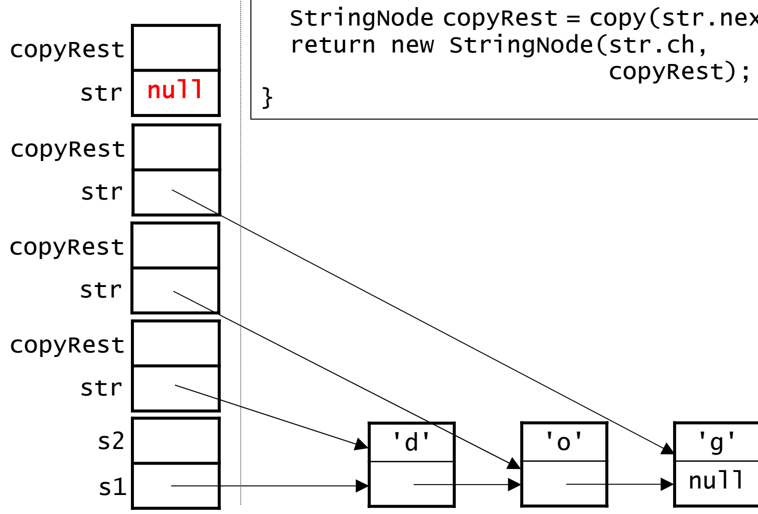


Tracing copy(): the base case

```
public static StringNode copy(...) {
  if (str == null) {
      return null;
  }

  StringNode copyRest = copy(str.next);
  return new StringNode(str.ch,
                          copyRest);
}
```



Tracing copy(): returning from the base case

```
public static StringNode copy(...) {
  if (str == null) {
      return null;
  }

  StringNode copyRest = copy(str.next);
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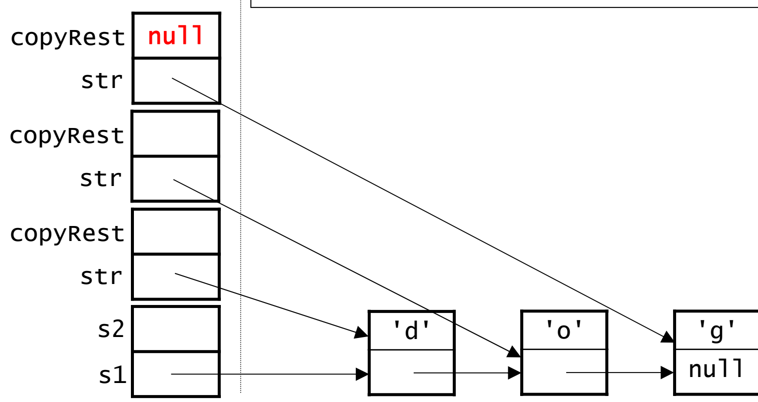
}
```



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}
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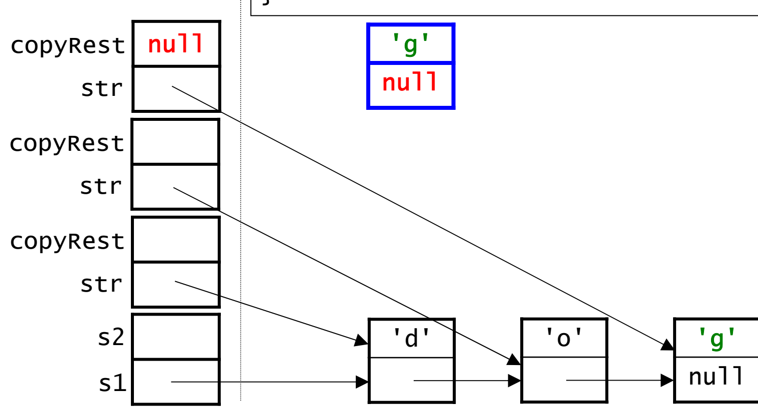

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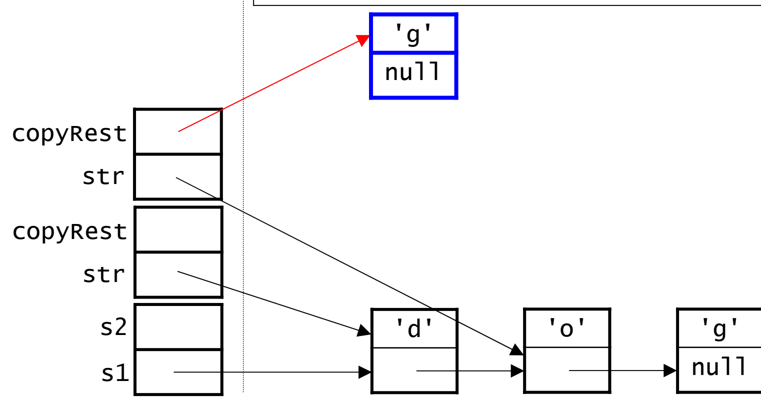
}
```



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```
public static StringNode copy(...) {
  if (str == null) {
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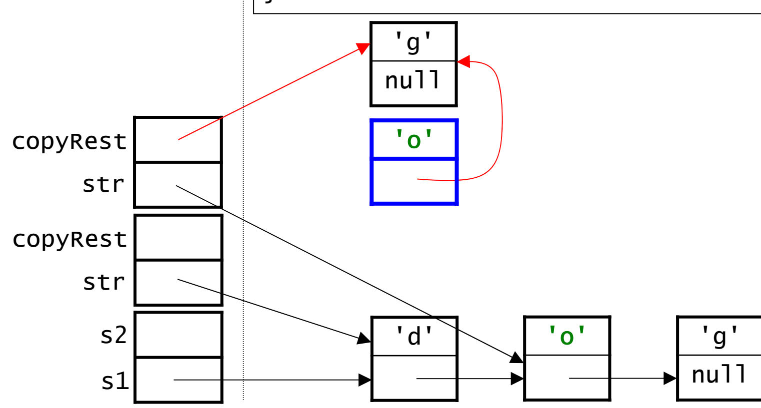
  }

  StringNode copyRest = copy(str.next);
  return new StringNode(str.ch,
                        copyRest);
}
```



Tracing copy(): returning from the base case

```
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  if (str == null) {
      return null;
  }

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  return new StringNode(str.ch,
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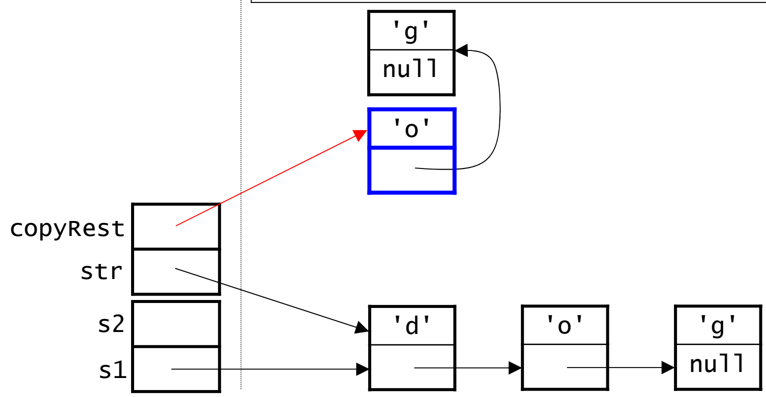
}
```



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```
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}
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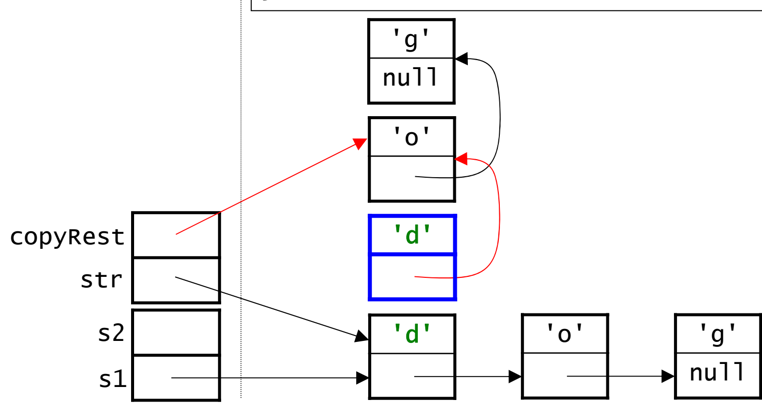
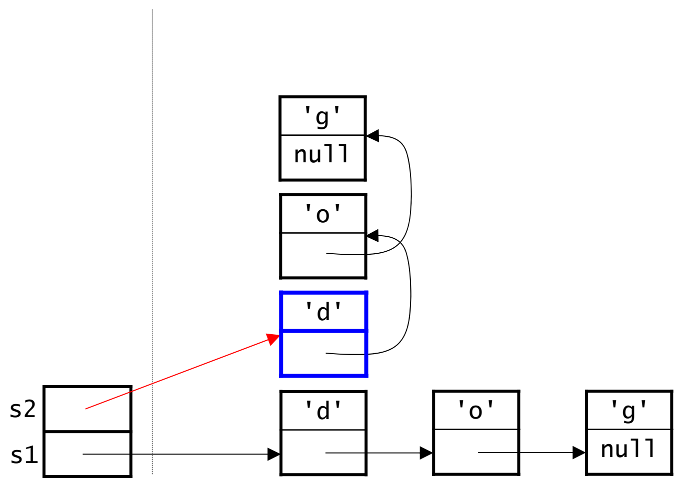


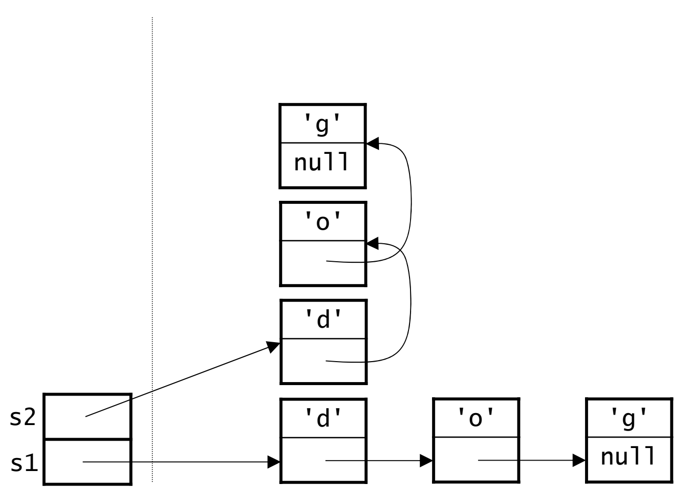
Tracing `copy()`: returning from the base case

- From a client: `StringNode s2 = StringNode.copy(s1);`



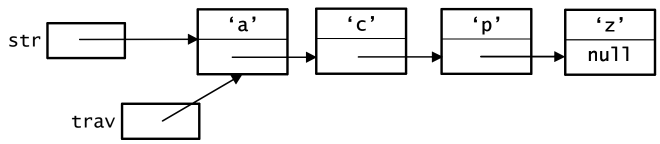
Tracing `copy()`: Final Result

- `s2` now holds a reference to a linked list that is a copy of the linked list to which `s1` holds a reference.



Using a "Trailing Reference" During Traversal

- When traversing a linked list, one `trav` may not be enough.
- Ex: insert `ch = 'n'` at the right place in this *sorted* linked list:



- Traverse the list to find the right position:

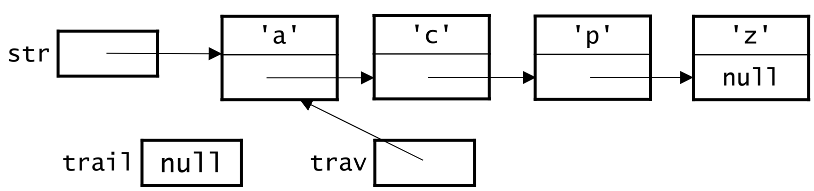
```
StringNode trav = str;
while (trav != null && trav.ch < ch) {
    trav = trav.next;
}
```

- When we exit the loop, where will `trav` point? Can we insert `'n'`?
- The following changed version doesn't work either. Why not?

```
while (trav != null && trav.next.ch < ch) {
    trav = trav.next;
}
```

Using a "Trailing Reference" (cont.)

- To get around the problem seen on the previous page, we traverse the list using two different references:
  - `trav`, which we use as before
  - `trail`, which stays one node behind `trav`




```
StringNode trav = str;
StringNode trail = null;
while (trav != null && trav.ch < ch) {
    trail = trav;
    trav = trav.next;
}
// if trail == null, insert at the front of the list
// else insert after the node to which trail refers
```

Using a "Trailing Reference" (cont.)

- To get around the problem seen on the previous page, we traverse the list using two different references:
  - `trav`, which we use as before
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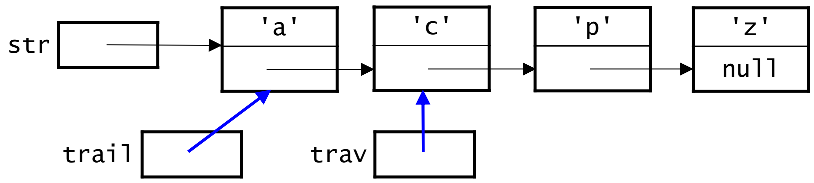


```
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}
// if trail == null, insert at the front of the list
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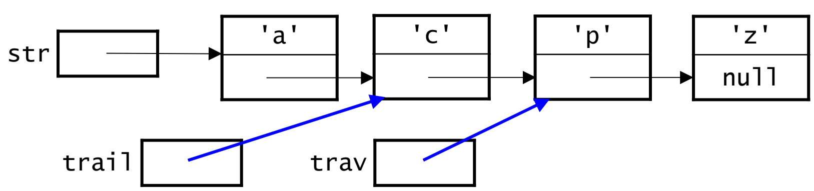


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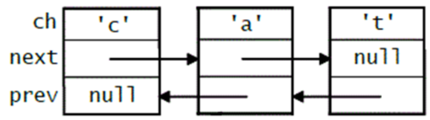
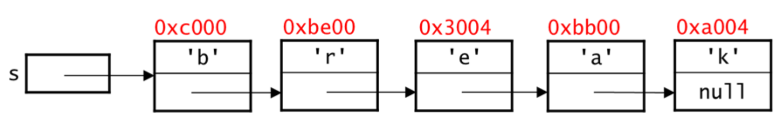
StringNode trav = str;
StringNode trail = null;
while (trav != null && trav.ch < ch) {
    trail = trav;
    trav = trav.next;
}
// if trail == null, insert at the front of the list
// else insert after the node to which trail refers
```

Doubly Linked Lists



- In a doubly linked list, every node stores *two* references:
  - `next`, which works the same as before
  - `prev`, which holds a reference to the previous node
    - in the first node, `prev` has a value of `null`
- The `prev` references allow us to "back up" as needed.
  - remove the need for a trailing reference during traversal!
- Insertion and deletion must update both types of references.

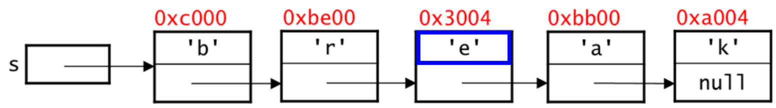
Find the address and value of `s.next.next.ch`



Extra practice!

	address	value
A.	0xbe00	'r'
B.	0x3004	'e'
C.	0xbb00	'a'
D.	none of these	

Find the address and value of `s.next.next.ch`



- `s.next` is the `next` field in the node to which `s` refers
  - it holds a reference to the `'r'` node
- thus, `s.next.next` is the `next` field in the `'r'` node
  - it holds a reference to the `'e'` node
- thus, `s.next.next.ch` is the `ch` field in the `'e'` node
  - it holds the `'e'`!

	address	value
A.	0xbe00	'r'
B.	0x3004	'e'
C.	0xbb00	'a'
D.	none of these	